POSITION REFERENCING AND CONSISTENT WORLD MODELING FOR MOBILE ROBOTS *

Raja Chatila and Jean-Paul Laumond

Laboratoire d'Automatique et d'Analyse des Systemes du CNRS
7, Avenue du Colonel Roche 31077 Toulouse Cedex, France

Abstract

In order to understand its environment, a mobile robot should be able to model consistently this environment, and to locate itself correctly. One major difficulty to be solved is the inaccuracies introduced by the sensors. The approach proposed in this paper to cope with this problem relies on 1) defining general principles to deal with uncertainties : the use of a multisensory system, favo ring of the data collected by the more accurate sensor in a given situation, averaging of different but consistent measurements of the same entity weighted with their associated uncertainties, and 2) a methodology enabling a mobile robot to define its own reference landmarks while exploring its environment. These ideas are presented together with an example of their application on the mobile robot HILARE.

One of the specific characteristics of mobile robots is the complexity of their environment. They have to cope with a wide qualitative and quantitative variety of objects, either as obstacles to be avoided or items to be manipulated. It is therefore essential that a mobile robot have the ability to build and use models of its environment that enable it to understand its structure. This is necessary in order to plan paths and trajectories, at least, but also to understand orders, and to plan and execute tasks.

The problems addressed in this paper are crucial when dealing with real-world mobile robots as opposed to simulated ones. They are the problems of : 1) constructing and maintaining an **accurate enough** environment model (of objects and space), that remains **consistent** as the robot explores new areas or sees again regions that are already modeled, and, 2) knowing its own position in this environment. The general problem concerns an unstructured environment, and the robot should be able to construct its references by itself. Imprecise mapping is an important issue in current mobile robot research[2].

A major difficulty stems from the fact that the machine perceives its environment and keeps track of its position through sensors that are inevitably imprecise. It is clear that these imprecisions will lead to a model degradation, and the identification of objects and areas already known and modeled would be in general impossible. Robot navigation would then be limited to the directly seen local environment.

To solve this difficulty, the error on the information about objects and robot position should be modeled for the concerned sensors. Our approach will be based on using two sensory devices measuring objects position (by vision or ranging), and robot position by trajectory integration. Based on the error models, we propose a method to improve the overall accuracy of the environment model on one hand, and of the robot position on the other hand. This method is partly implemented on the mobile robot HILARE[3 4] and will be illustrated on an example.

1. World models and position referencing

1.1. World Models

In the scope of the mobility function, three kinds of world models are necessary: a geometrical model, a topological model, and a semantic model:
A. The geometric model is directly deduced from perception data. We deal here with objects that are to be considered as obstacles to be avoided or approached, and not smaller items to be manipulated. In the case of our experimental robot HILARE for example, objects have polyhedrical shapes and are ground projected to obtain a two-dimensional model. This model basically contains the position in an absolute coordinate frame of the object projection vertices. It is the first environment representation and will be manipulated to deduce the topological and semantic models. It is it that will have to be updated and maintained as accurate as possible. The objects are referenced in an absolute coordinate system and we will see that vertices and objects will have an associated uncertainty in this frame.
B. Topological model. Space structure understanding is based on its topology, which expresses its intrinsic properties at a given scale. The topological model is a hierarchy of several levels. Let us define the concept of **place** as an area that is a functional or topological unit. For example, a room, a corridor are topological units, a workstation is a functional unit. The concept of place is defined at several levels: a room, a storey or a whole building are places. In an outdoors environment, the concept of place may also be defined with

*These issues are discussed in more detail in <1>.

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specific characterizations. Connectors link places: doors, corridors, stairways or elevators are for instance indoors connectors. Some places may also be connectors themselves. The topological models are the connectivity graphs of places, the arcs being the connectors, at each level.
At the lowest - or more detailed - level, the topology of space is deduced from the geometrical model by structuring the empty space[3]. Space structuring is obtained by constructing convex polygons, called cells, based on obstacle edges and vertices (fig. 1). A cell is a place, and a set of cells may be a place if it is a topological or functionnal unit. The cell common edges are connectors. The cell connectivity (adjacency) graph constitutes the first topological representation of space structure. As the robot explores its environment and discovers new areas, space structuring is performed, and the graph is augmented, and modified. Another graph the nodes and arcs of which are respectively the cell common edges, and the cells is also constructed. Path search is performed on this latter graph because it permits to express better the associated cost[5]. This space structuring scheme can also be found in another paper[2] where the places are "freeways" and "meadows". More abstract topological models are built by structuring the cell graph; the components thus obtained can be labelled[6] to introduce a semantic contents (doors, rooms, corridors,...).
C. Semantic model. This symbolic model is to be manipulated at high decision-making levels and should contain information about objects and space properties and relationships. In the case of HILARE, this model is currently partially distributed among the geometric and topological models, through object and graph labellings and attributes. On this subject, our work has mostly concentrated for the time being on introducing meanings into the topological models.

Figure 1 shows a robot indoors environment consisting of several rooms containing obstacles, the cell structuring, and the topological models at various levels. Semantic labelling was produced automatically by the system after decomposition of the global cell-graph and using labelling rules.

1.2. Position referencing

A mobile robot is not linked, like a manipulator, to a fixed point. This simple fact makes robot position finding much more complicated because there is not a permanent reference or a simple calibration procedure. Three means can be defined for mobile robot position referencing:

A. **absolute position** referencing by using fixed and known beacons. To do so, it is necessary to cover the whole robot world with a beacon system so that it can compute its position at any moment. This means that the robot's environment is structured, and this is an important limitation. Robot position errors are, in this solution, only related to the beacon and borne measurement systems accuracies.

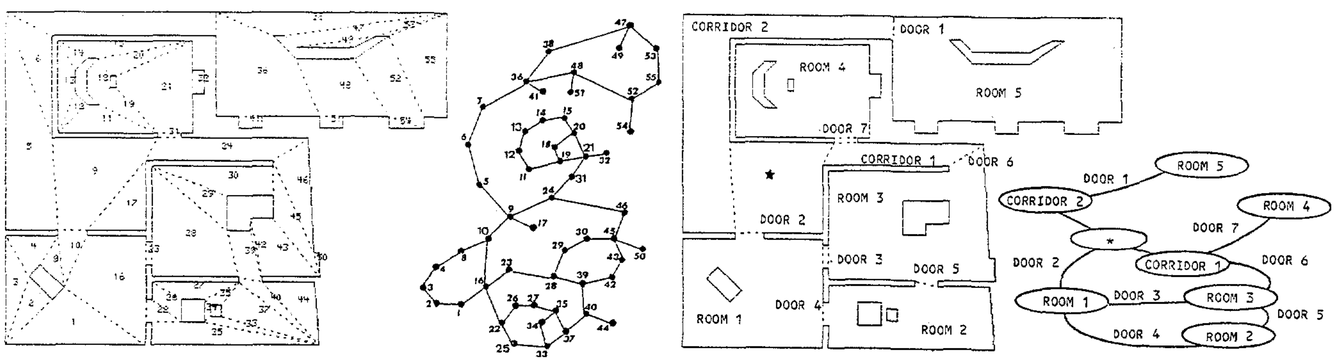
B. **trajectory integration** by using some odometric or inertial system that provides position and orientation without any external reference. Here, errors are cumulative and result in a position drift. It is therefore necessary to periodically adjust the position by another means.

C. **relative position** referencing with respect to objects or places. This referencing is more general and complex, and requires an environment model. It makes use of the tools developed in this paper. The robot defines by itself its references which are either characteristic features of the environment or objects known with good accuracy.

These three referencing means are complementary on a mobile robot and are developed on HILARE.

1.3. Model construction and degradation

World models are generally built gradually by the robot itself as it moves and discovers new areas. As long as only **new** parts of space are discovered, there is no possibility to find out and correct any errors, in the model being built, that are due to robot position drifts. But when the robot is to perceive again already known and modeled regions, inconsistencies may occur, and this correction becomes necessary, and sometimes possible.



Cell structuring (place=cell)

1st topological model cell graph

Space structuring deduced from cell-graph decomposition. (place=door, room, corridor) Area * is not labelled by the current set of rules.

Topological model with semantic labels

FIGURE 1

At every moment, the robot possesses:
a. a current environment model containing the geometrical, topological and semantic knowledge acquired so far; the geometrical model is related to an absolute reference frame,
b. a knowledge about its position and attitude, and
c. a robot-centered geometrical model of the local environment perceived at that moment.

It faces the central problems of updating the models of (a) using the knowledge of (b) and (c), and of correcting the information in (b) if possible.

These problems are trivial for a perfect robot using known geometrical transformations corresponding to robot movements. For a real-world robot, errors at several levels of the perception and position measurement processes, after motions that are themselves inacurrate, are to be taken into account, and maintaining a consistent model becomes more complicated.

2. Multisensory approach and measurement errors

2.1. Multisensory approach

Space modeling and robot path planning require a knowledge about object form and location. A visual-like 3D perception system is necessary for this purpose. In addition, in the absence of any a priori known external reference, robot location can only be done through trajectory integration. Therefore, two different perception means are necessary in general. On the other hand, the visual-like system should be sophisticated enough to provide information in a variety of situations including large dynamics (proximity and long range sensing; large and small object modeling...), and quick response to external changes. Such specifications won't be met by a unique device; the robot should possess a multisensory system covering a wide range of signals and information. Each of these sensors has a range where it is most adapted. Some information may be acquired by two or more different devices.

In our experimental example, the two sensory devices are a laser range finder that provides a depth map for model building, and optical shaft encoders on the drive wheel axis for trajectory integration (this is not a general odometry system, but is sufficient to illustrate our approach).

2.2. Dealing with errors

The data acquired by the sensors are uncertain. Errors are produced at 1) measurement, because of the considered sensor characteristics (noise, accuracy,...), and 2) processing by the data interpretation algorithms.

At measurement phase, the errors are of two types: systematic errors and random errors modeled by a gaussian. We suppose that systematic errors, due to a bad sensor positionning for example, can be identified by a calibration method. We will deal only with random errors; the gaussian's parameter are determined experimentally. We will always use a gaussian model of the error, even if it is necessary to consider an upper bound of the error to do so. Redundant measurements of the same entity by two different sensors, or by the same sensor in two different experimental conditions, are generally different, and may be inconsistent. We suppose that measurements are consistent in the scope of this paper (this means that we have a good model of the error). If inconsistencies should occur, they can only be detected here but should be dealt with at another decisional level controlling and organizing the sensory system and possessing more global knowledge.

Our approach will be to consider an average value of consistent redundant measures if the accuracies (standard deviations) have the same magnitude, and the most accurate value (smallest standard deviation) otherwise. Appendix 1 presents these notions with more detail.

3. General methodology

The methodology indroduced here provides a general approach for dealing with errors in the space representation models being built by a mobile robot. It has been developed using HILARE as an experimental support. The objects in the robot's environment have polyhedral shapes; this makes our approach easier to experiment without a loss of generality. The principal ideas that will be developed are:
1) to associate a frame of reference with every new discovered object. The point here is to reference an object's vertices locally to this frame, and the whole object to the absolute frame of reference. This results in keeping the local and relative relations between object vertices and edges.
2) to introduce a prediction model containing the portion of the known environment that could be seen after a movement, in order to match more efficiently the current model with the perception model.
3) the notion of continuity in seeing an object at two consecutive perceptions from two different locations.
4) after correcting the current position of the robot by matching the prediction model with the perception model, to modify the earlier positions accordingly in order to update the world model acquired so far, taking into account the new position changes.

We will rely on the following principles:
a) select the most accurate sensor if possible, and
b) weight-average consistent measures of the same entity, taking into account the associated uncertainties.

It might be helpful to read section 4, wherein an illustrative example is provided, in parallel with the following sections.

3.1. Object reference frames

Each object has its associated frame of reference. This frame is introduced when a part of the object is discovered for the first time. The frame is linked to an object element that is best characterized (a vertex and two edges rather than one edge for example, if we consider geometrical features only. Other aspects (color, semantic knowledge,...) can also be considered if the sensors provide this information and if the models represent it). If the element is not reliable enough, for example if the only known part of the considered object is an edge the extremities of which are not actual vertices but result from the projection along the lines of sight of other object vertices, then the associated frame may be temporarily linked to that element and changed after other perceptions (figures 2a and 3b).

Elements of an object are referenced in its linked frame with their associated perception uncertainties. The frame itself is referenced in the absolute frame with the associated uncertainty on the robot position at the moment it is constructed. If the object's elements to which the frame is linked are to be modified by further perceptions (average of several perceptions for example), then the uncertainty on the frame itself will be modified accordingly, taking into account the uncertainty on robot position at each of these perceptions.

Let E_p be the uncertainty resulting from the perception error on the element on wich is based the object linked frame, and E_o the robot position (location and orientation) uncertainty. E_o results from the odometer error which is cumulative (with the travelled distance) and is decomposed into an angle error (robot orientation) and a location error of a point attached to the robot. The object frame's uncertainty in the absolute frame is a combination of E_p and E_o. After a movement, the robot position uncertainty is E'_o, larger than E_o, and let E'_p be the uncertainty due to perception on a newly seen element of the object. In the absolute frame the global uncertainty on this element is a combination of E'_o and E'_p, but this does not take into account the fact that the new element has geometrical relationships with the parts of the object already known. The object linked frame expresses these relations; the new element has a smaller uncertainty in it: E'_p combined with the differential uncertainty between the two considered robot positions instead of the global uncertainty E'_o.

3.2. Continuity

A part of an object that is newly discovered at perception P_{n+1} is said to be in **continuity** with the current environment model M_n if this part was seen at the previous perception P_n. This relation conditions the creation of a new object frame, by considering the relation's transitive closure. It also enables to reduce the computations for deducing the prediction model (see §4).

3.3. Updating the world model

Prediction model. For a perfect robot (one without motion and measurement errors), the perception model (what is seen at perception P_{n+1}) can be integrated to the current model M_n (what the robot has discovered so far, from perception P_1 to perception P_n) by using simple reference frame transformations based on the robot's movement bet ween perceptions P_n and P_{n+1}.

For a real robot, with imprecise models and movements, the problem is more complex. The prediction model is first computed as if the robot was perfect, i.e., by supposing that the movement between R_n and R_{n+1} is known with a perfect accuracy. It is given easily through the transformation of M_n, by the considered robot movement, and by the computation of the visibility polygon from the new position. This partitions M_n into three parts (fig. 2b):

- the part that **should** be seen at P_{n+1}: SM_n.
- the part that **should not** be seen at P_{n+1}: NM_n.
- the part about which no decision can be made: IM_n.

Note that semantic knowledge may be used to infer geometrical information for this partition.

This partition is used to match the perception model at R_{n+1} with M_n.

Identification of already known areas. At perception P_{n+1}, the robot has the following knowledge:

- The model M_n, partitioned as mentioned above (SM_n, NM_n, IM_n). In this model, each object is referenced to its linked frame, its parts having uncertainties due to perception inaccuracies, and the frame itself is referenced in the absolute frame with an associated uncertainty taking into account robot position imprecisions.
- The perception model PM_{n+1} and associated perception uncertainties.
- Robot position and orientation R_{n+1} in the absolute frame with a known accuracy.

Matching PM_{n+1} with SM_n uses the continuity relation between perceptions P_n and P_{n+1} and is based on our (experimental) hypothesis that, for short travel distances, we can neglect odometry errors. The parts of PM_{n+1} that are not in continuity with PM_n are tested for matching SM_n, by comparing the vertices coordinates, taking into account perception and odometry uncertainties, as well as topological information.

After matching the prediction model with the perception model, the new measures of object parts that were already known are taken into account to improve the model, if their accuracy is better than the model's, in the object linked frame. A weighted average between the values in P_{n+1} and in SM_n is thus computed for those parts, and included in the model M_{n+1}.

If the accuracy of the object model is better than the new measures, i.e., if the linked frame in the model is more accurately referenced to the absolute frame than the current robot position, then we can correct the position of the robot relatively to the model (§ 3.4.).

Updating the model with new perceptions. Integration to the model of newly seen object parts that are in continuity with perception P_n is done by referencing them to the object-linked frame with their perception uncertainties combined with odometry uncertainties between R_n and R_{n+1}.
New discovered object parts that are not in continuity are added to the model after creating associated frames based on a characteristic element if possible (§ 3.1.), and the frame itself is referenced in the absolute frame with the robot's position associated uncertainty.

Free space portions that are not based on obstacle edges or vertices (i.e. regions bounded by lines of sight and constituting unknown "islands" in midst of known areas in the model) can be discovered by the new perception. Detection of such situations is done by computing intersections of the new lines of sight with the island's bounderies, taking into account the inaccuracies on the lines of sight extremity.

3.4. Robot position correction

Matching the current model with the perception model provides a means for correcting the robot's position.

Let us consider a portion of the environment that is in the current model and in the perception model. In our 2D polygonal environment, for example, this portion is a set of polygon edges. To correct robot position, this portion's accuracy i.e., the accuracy of the linked frames position in the absolute frame, should be better in the current model than the robot's position accuracy in the absolute frame. As mentioned in § 3.3., matching the model with the perception is done by comparing the vertices coordinates and edge angles in the case of a polygonal environment. But, because of perception inaccuracies, there generally does not exist an isometry that transforms exactly the figure in the perception model to the figure in the current model (Fig. 5c). But we can find one that matches the two figures **globally** (appendix 2). This isometry can then be applied to the current robot position. The new position thus obtained has the same global uncertainty as the part of the model that was used to compute the transformation.

3.5. Fading

After correcting the robot's current position, the previous positions must be examined for possible corrections also. We have therefore to transmit this correction backwards; but because odometry errors are cumulative, this transmission is not uniform. We introduce a "fading" function to do so:
Let us suppose that the robot accomplished a trajectory including a sequence of positions $R_1,\ldots,R_n$ where perceptions were made. And let us suppose that the correction took place at position R_n, where the associated uncertainty is u_n. The new corrected position is R''_n and the associated uncertainty u''_n ($u''_n < u_n$). Since position error is cumulative, there exists a position R_k such that u_k is of the same order of magnitude as u''_n. This position corresponds generally to the creation of the coordinate frames of the objects that were used for correcting position R_n. The fading function transforms the positions between R_n and R_k into other positions $R''_{n-1}\ldots R''_k$ obtained by just operating on $R_{n-1}\ldots R_k$ the same transformation that was operated on R_n. But uncertainties grow backwards on positions R''_i thus obtained: $u''_n < u''_{n-1} <\ldots< u''_k$, whereas $u_n > u_{n-1} >\ldots> u_k$. The actual position (or its best model R'_i) is obtained by computing a weighted average of every R_i with its corrected value R''_i whenever u_i and u''_i are of the same order. Otherwise, the most accurate value is selected according to our general principle of §§ 2.2. and 3. Figure 5e shows an example of robot position correction and illustrates the fading function.

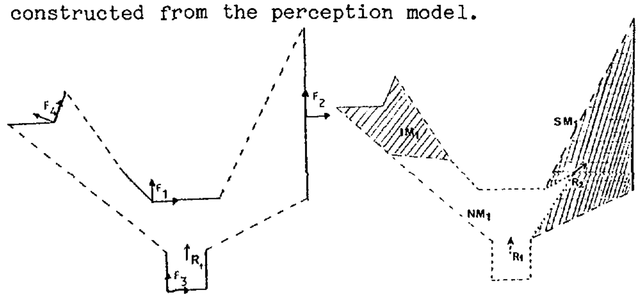
3.6. World model correction

All object-based frames that were built between R_k and R_n have uncertainties due to robot position errors. After correcting positions $R_k,\ldots,R_n$ and obtaining more accurate values $R'_k\ldots R'_n$, we have to correct the world model accordingly. Therefore, the concerned frames parameters (those built between R_k and R_n) are modified in the absolute frame, as well as their associated uncertainties. This correction might lead to the deformation of some objects. Indeed, some objects might have rather long perimeters (for instance, this the case for walls as in our example) and it is necessary to associate more than one frame with the same object, to keep a more accurate model. Correcting these frames differently will result in object deformation.

4. Example

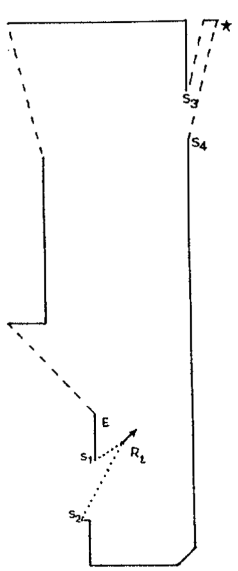
This section is mainly composed of a sequence of figures that help to understand the ideas exposed in the paper. The robot is in an unknown indoor environment (two rooms with unknown obstacles, and workstations WS1 and WS2). The robot's task is to feed workstation WS2 with parts that are in a storage place SP (fig.6). Its initial position is at a battery charging station. Its initial knowledge is its position and orientation in an absolute world frame, as well as the positions of the storage place and the workstation in this frame. The robot will rely in this example on two sensors: a laser range-finder for environment acquisition, and an odometer for trajectory integration. We suppose also, according to our multisensory approach, that it possesses a proximity sensing system for docking to the storage place and the workstation, for example.

Robot perceptions will be "goal-oriented" since its aim is not to explore its world, but to perform a given task.
Figure 2a. represents the first (panoramic) perception. Four different connected components are perceived and four associated frames are built. Frame F_2 cannot be attached to a characteristic element; its origin has been arbitrarly fixed as the middle of the edge. The first model is constructed from the perception model.

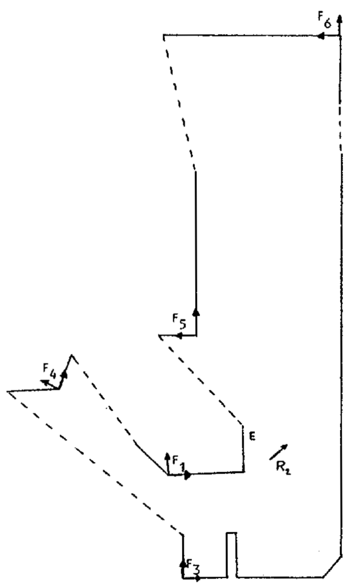


2a: 4 object reference frames 2b: Prediction model at R2

Figure 2b. shows the prediction, model after the robot movement from R_1 to R_2. Perception is limited to the cone (S_1,S_2) to guarantee continuity. In the prediction model, the grid area is the part to be certainly seen (SM_1), and the hachured area the part that might be seen (IM_1). Figures 3ab show the perception and deduced models at R_2. Continuity relations between P_1 and P_2 allow to reference the new dicovered edge (E) in F_1, and to merge F_2 with F_3. F_5 and F_6 are new object frames. The starred edge is not taken into account in the deduced current model because 1) it is computed from weak information (small number of points), and 2) its distance to the robot makes these measures too imprecise. But the information that an opening between S_3 and S_4 exists is kept.

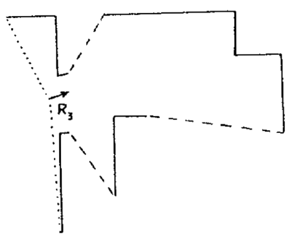


3a: Perception P2

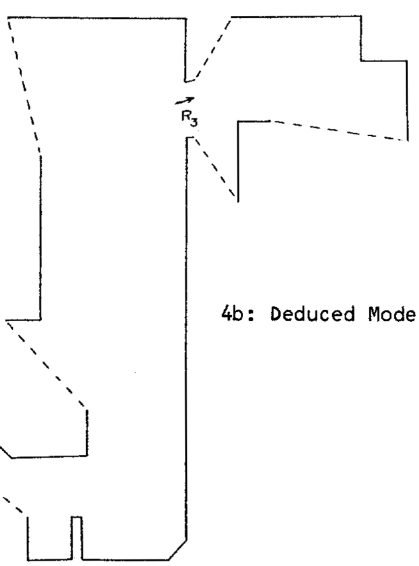


3b: Deduced model

Figures 4ab. represent the perception and deduced models after perception P_3.



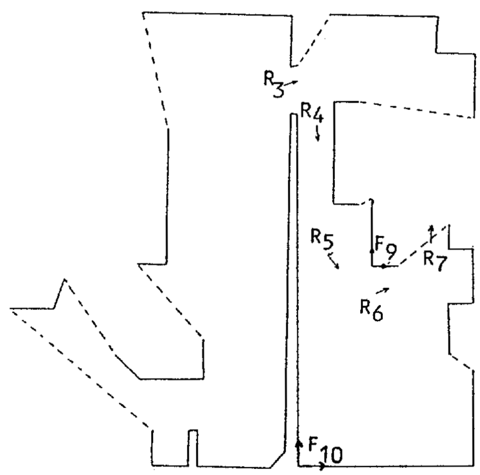
4a: Perception P3



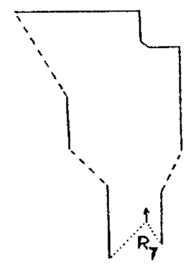
4b: Deduced Model

The basic routines of prediction, perception and model deduction are carried out at positions R_4, R_5, and R_6.

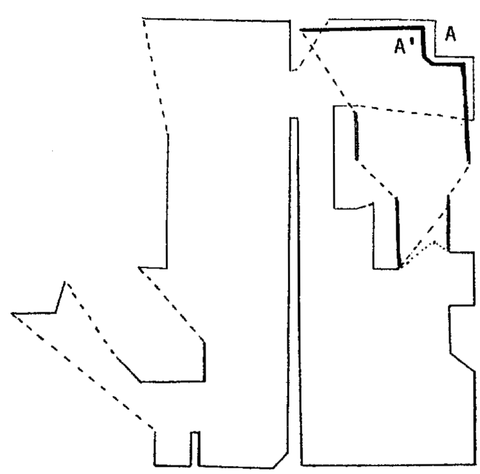
Figure 5a. shows the current model at the seventh perception P_7, and robot positions R_4,R_5,R_6 and R_7. Figure 5b. shows the perception model and figure 5c. the superposition of the current and perception models based on the measured robot position. Matching the regions common to the current and perception models (A and A' in fig. 5c.) gives the parameters for the correction of robot position R_7 (fig. 5d.). The reference region A was first modeled at perception P_3 (robot position R_3). Thus, positions R_4, R_5 and R_6 will also be corrected, with a fading function for uncertainty computing. The new position R''_7 will have the same uncertainty as R_3. Figure 5e. shows the final positions R'_7, R'_6, R'_5 and R'_4. $R'_7=R''_7$ ($u''_7 < u_7$). Position R'_6 is also R''_6, deduced directly from R''_7 ($u''_6 < u_6$ also), whereas R'_5 results from averaging R_5 and R''_5, u_5 and u''_5 being of the same order. R'_4 is R_4, u_4 being too small compared to u''_4. Frame F_9 position, built at perception P_5 (robot at R_5, fig 5a.), is corrected accordingly in the absolute frame .



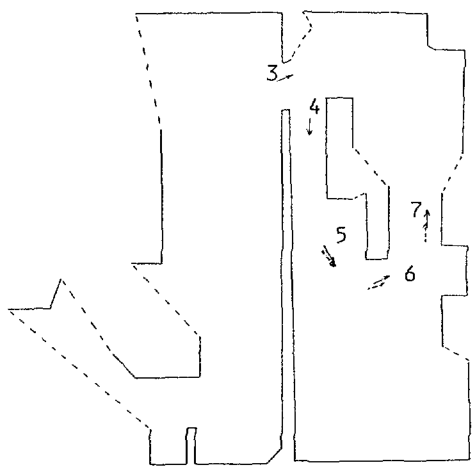
5a: Current model P7



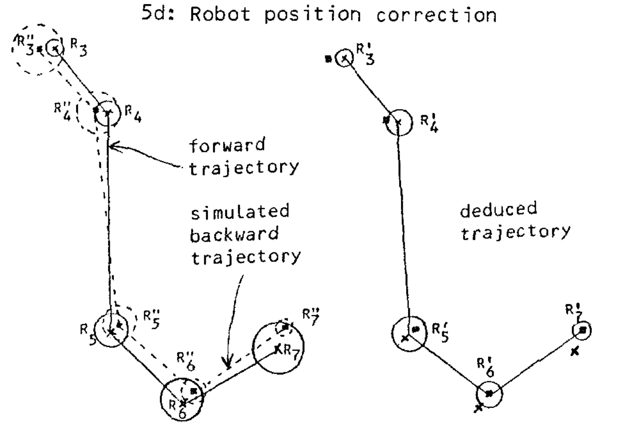
5b: Perception P7



5c: Superposition of M7 and P7



5d: Robot position correction



5e: Fading
(orientation correction is not represented)

At R_7, the robot is able to take the desired object from the storage station. Note that if this station's position is a priori known, the robot could use it to correct its position also.

In figure 6, the trajectory to position R_8 is shown. It is within the known part of the environment. It is important to note that, due to model inacurracies and robot position drift, the door crossing for example might be critical. Quick perceptions in order to check the model (and not to acquire new information) could be done between R'_7 and R_8. In the case of HILARE, an ultrasonic system is used for close range local obstacle avoidance. At R_8, no continuity relation can be defined but a robot position correction can be made using the global model.

At position R_{10} -the final goal to be reached- (figure 6) a position correction can also be done if the workstation's position is known. On this figure is shown the final model with the associated uncertainties on robot and object frame positions.

5. Conclusion

Dealing with errors is a difficult but important issue in robotics. We presented an approach, relying on some basic general ideas, to solve the problem facing a mobile robot with imprecise sensors trying to model its environment and to locate itself in it, without any external reference. The presented ideas are currently being implemented to be used on the mobile robot HILARE, which served as a basis to develop them, after several experimentations of its navigation and multisensory systems.

This error processing is only one aspect of the problems to be solved when a mobile robot faces real-world situations. Other problems such as taking into account events not explicitely foreseen, or robot action based on uncertain models *such as those produced by considering measurements* errors, are to be dealt with by another decisional level[7].

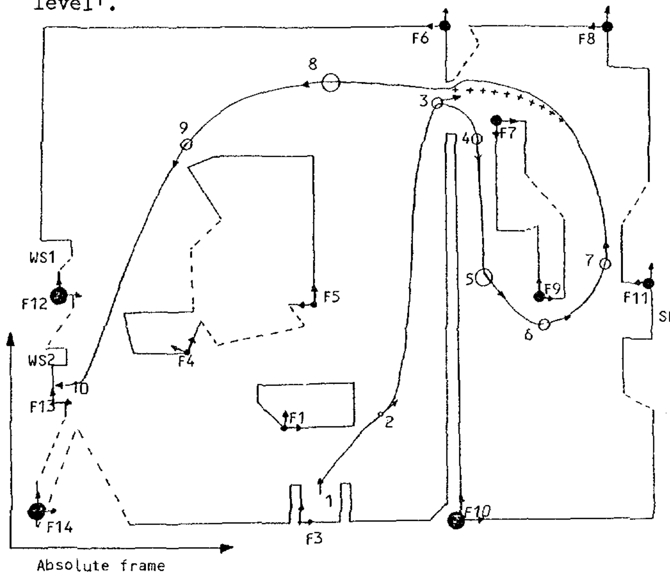


FIGURE 6: The final model with the associated errors on robot and frame position (the orientations errors are not represented)

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REFERENCES

<1> R. Chatila, J-P. Laumond, Controle et mise a jour de la position et du modele de l'espace d'evolution d'un robot mobile. Technical note LAAS-84059, Toulouse, Aug. 1984.

<2> R. A. Brooks, Aspects of mobile robot visual map making. 2nd ISRR, Kyoto, Japan, Aug. 1984.

<3> G. Giralt, R.P. Sobek, R. Chatila, A multilevel planning and navigation system for a *mobile robot; a first approach to HILARE.* 6th IJCAI, Tokyo, Japan, Aug. 1979.

<4> G. Giralt, R. Chatila, M. Vaisset, An integrated navigation and motion control system for autonomous multisensory mobile robots. 1st ISRR, Bretton Woods, NH, 1983.

<5> R. Chatila, Path planning and environment learning in a mobile robot system. 2nd ECAI, Orsay, France, July 1982.

<6> J-P. Laumond, Model structuring and concept recognition: two aspects of learning for a mobile robot. 8th IJCAI, Karlsruhe, Germany, Aug. 1983.

<7> G. Giralt, Research trends in decisional and multisensory aspects of third generation robots. 2nd ISRR, *Kyoto, Japan, Aug. 1984.*

APPENDICES

1 - UNCERTAINTY PROCESSING.

The measurement errors considered here are modeled by a gaussian (0,u). In some cases, an upper bound of the error is adapted to meet this model. Let X be the measured value of a quantity X^*. We have, according to the gaussian model :

$P(|X-X^*| < u) \approx .68$, $P(|X-X^*| < 2u) \approx .95$

where P(a) reads probability of a.

Two measures X_1 *and* X_2, *of a same quantity* X^*, with associated uncertainties u_1 and u_2, will be said to be **consistent** if $|X_1-X_2| < u_1 + u_2$.

The weighted average X of two consistent measures X_1 and X_2 is computed as follows:

$$X \approx \frac{u_1 u_2}{u_1 + u_2} \left(\frac{X_1}{u_1} + \frac{X_2}{u_2} \right)$$

Let u be the standard deviation of the gaussian modeling the error on X; the gaussian (0,u) is the linear combination of the gaussian (0,u_1) and (0,u_2), supposed to be independant :

$$u = \sqrt{2}\,\frac{u_1 u_2}{u_1 + u_2}$$

Let $u_1 < u_2$. The averaging operation produces a better accuracy if $u < u_1$, i.e. :

$$(\sqrt{2} - 1)\, u_2 < u_1 .$$

Therefore two uncertainties u_1 and u_2 will be said to have the same magnitude if

$$(\sqrt{2} - 1)\, u_2 < u_1 < u_2.$$

The adopted value of a quantity X^* will be :

- a weighted average X if u_1 and u_2 have the same magnitude,
- the value having the smallest uncertainty otherwise.

This method produces a value that has always the best accuracy.

2 - MODEL AND PERCEPTION MATCHING.

Because of accumulated perception and robot position errors, there exists not, in general, an isometry that establishes an exact transform between the model and a new perception of the same objects or object elements. We develop herein a solution to this problem in the planar representation. The objects have polygonal shapes. In a first step, we try to establish a bijection between the set E of vertices in the model and the set E' of vertices in the perception. But due to errors, a given element in reality might have different representations in the model M_n and in the perception P_{n+1} (figure A). Therefore, there exists not in general a bijection between E and E'. To overcome this difficulty, we proceed as follows:

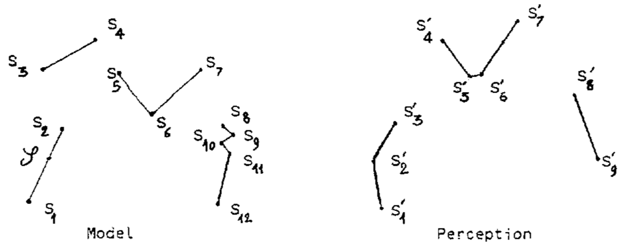

FIGURE A

If a vertex S in the model, with uncertainty u, and a vertex S' in the perception, with uncertainty u', have consistant positions in the absolute frame (Distance(S,S') < u + u') , then S and S' are matched together, and we write C(S;S'). We might have also:

1) multiple correspondance: $C(S;S'_1,\dots,S'_k)$ or $C(S_1,\dots,S_m;S')$ (for example $C(S_6;S'_5,S'_6)$ or $C(S_8,S_9,S_{10},S_{11};S'_8)$). In this case, the barycenter of $S'_1,\dots,S'_k$ (respectively $S_1,\dots,S_m$) is computed, with the associated uncertainties as weights (appendix 1), and matched with S (respectively S').

2) a vertex that has no correspondant in the other set but should have. This is the case for example of a vertex produced by a perception of an edge as two edges (S'_2 in the figure). In this case, a vertex is added in the incomplete set to match it, taking into account the local relationships :

$$\frac{S_1\mathcal{S}}{S_1S_2} = \frac{S'_1S'_2}{S'_1S'_3}.$$

E and E' are thus transformed into two other sets of vertices F and F'. The above matching operations establishes a point-to-point mapping between F and F'.

The second step consists in finding the global isometry that superposes F and F' "to the best", respecting the point-to-point mapping. The isometry is defined by a vector V(X,Y) and an angle Θ. We have for each vertex $S_i(x_i,y_i)$ in F:

$$\begin{pmatrix} x''_i \\ y''_i \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\Theta & -\sin\Theta & X \\ \sin\Theta & \cos\Theta & Y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$$

The set F" thus obtained should match F'. The criterion of best match is the minimization of

$$\sum_i w_i \,\mathrm{Distance}(S''_i,S'_i),$$

the weighted sum of the distance between corresponding vertices. The weight w_i takes into account the uncertainties on S_i and S'_i in order to favor the more accurate vertices.

Another solution to model and perception matching that is being investigated is to find the isometry that 1) superposes the barycenters of sets E and E' (computed with the vertices associated uncertainties as weights), and 2) minimizes the area of the polygon defined by E and E' after superposing the barycenters.